# Effect of spurious resonant modes on the operation of radial mode piezoelectric transformers

Jack, Forrester, University of Sheffield, United Kingdom, jforrester1@sheffield.ac.uk
Jonathan, Davidson, University of Sheffield, United Kingdom, jonathan.davidson@sheffield.ac.uk
Martin, Foster, University of Sheffield, United Kingdom, m.p.foster@sheffield.ac.uk

## Abstract

This paper analyses both the intentional and unwanted resonant modes of radial piezoelectric transformers (PTs). Initially, finite element analysis is performed to discover the type of spurious modes prevalent is radial mode PTs. From this, analysis is performed on these modes and a relationship between resonant frequency and geometry is found. The effect of spurious modes on the efficiency of a typical PT is simulated using an equivalent circuit model in LTSpice and Simulink. A number of design rules are generated based on the findings. Results show that, in most cases and by careful design, spurious modes can be avoided.

## 1. Introduction

Piezoelectric transformers are seen by many as an attractive replacement for traditional magnetic transformers. High power densities, minimal electromagnetic interference, small size and high efficiency are some of the many advantages that PTs provide. They are already employed in a number of applications, such as compact fluorescent lamps and TVs. PTs are very attractive as part of a resonant power converters due to their inherent resonant tank and they therefore require minimal/no magnetic components to achieve zero voltage switching (ZVS) and efficient operation. Despite the advantages, designing PTs is a complex process and so there are currently only a small number of commercially available devices.

One area of the design process that causes issues is the interference caused by unwanted/spurious modes. Radial mode piezoelectric transformers are well known to operate best when vibrating longitudinally, as this provides the highest coupling factor [1]. Radial mode devices, like all types of PT, have a number of other vibration modes that can also occur and currently the primary method of detection for these modes is using finite element methods (FEM). These modes cause unwanted tension in the device, reducing power density and causing increased loss, thus lowering efficiency [2].

In this paper, the types of typical spurious mode in radial mode devices are derived, from this an understanding of the relation between the material properties and geometry, and the resonant frequency is analysed. The effect of spurious modes on the operation of these devices is investigated and a number of design rules are presented.

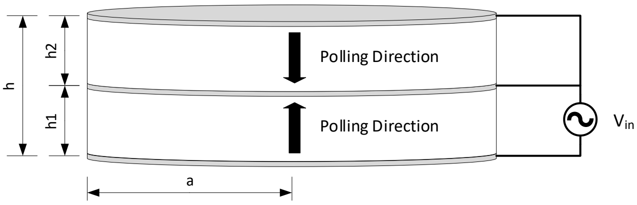


Fig 1 - Basic model of a radial mode PT (electrode areas are assumed infinitely thin)

## 2. Spurious vibration modes in radial mode piezoelectric transformers

In order to analyse the spurious modes that occur at frequencies near that of the radial vibration mode, their vibration shape must be determined.

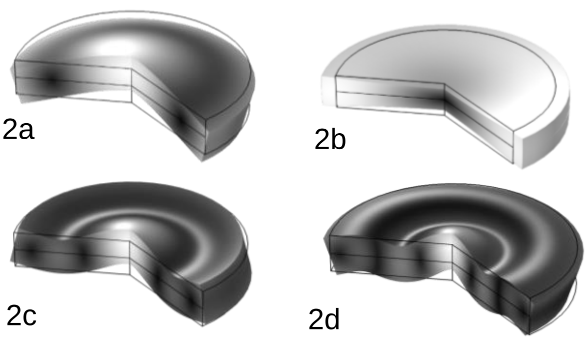


Fig 2 – Resulting displacement plots for the first 4 eigenmodes of a radial mode PT with a radius of 9mm and thickness of 3mm

An eigenfrequency study was performed in COMSOL on a simplified 2D axisymmetric model of a piezoelectric transformer, with a radius of 9mm, a thickness of 3mm and made from PZT-5H.

Fig. 2 (a-d) shows the first 4 mode shapes for this 2-layer device. From Fig. 2 it is apparent that the modes occurring around the radial vibration mode are a symmetrical mode, known as flexural vibrations. Flexural modes can be categorised by the number of nodal diameters (d) and nodal circles (c), written as (d, c). Fig. 2 (a,c,d) show modes (0,1), (0,2) and (0,3). Fig.2 (b) shows the first radial mode of this device.

This eigenfrequency study was then performed for a number of other devices with differing dimensions. The results of the study were largely similar, although the frequencies at which each mode occurred changed for each device. In all cases the modes occurring in close proximity to the radial mode were flexural vibrations, and they will be the focus of this paper.

## 3. Determining resonant frequencies of flexural modes

In order to optimise the design of a PT for minimum interference from spurious modes, the relationship between the geometry and material properties, and resonant frequency must be evaluated.

### 3.1. Elastic circular plate theory

Free vibrations of elastic plates have been a comprehensively examined topic over the last two centuries, with many exact and approximate solutions to the problem. Most notably, in 1850, Kirchhoff published a thesis on the theory of thin elastic plates, the work included in his thesis presented the foundations for solving this type of problem [3]. A wide range of authors have extended Kirchhoff's work to improve accuracy and include more complex boundary conditions [4]. Leissa published a reference book in 1969 that summarises the work done on this field and included some numeric solutions [5]. The majority of the work in this field has been based on 2D simplifications of the 3D problem, due to the complexity and lack of exact 3D solutions.

Extending this to piezoelectric plates adds additional complexity, as now the electrostatic potential must also be solved for. As a result, exact 3D solutions are currently not available [6]. Fortunately, approximating a piezoelectric plate as an elastic plate is valid in most cases [7]. In the following analysis, the PT will be approximated as a single disk with thickness 'h' and radius 'a', shown in Fig.1. The analysis will be based largely on elastic theory.

From Leissa [5], the classic equation for flexural vibration of a thin circular plate under Kirchhoff's assumptions is:

$$D\nabla^4 w + \bar{\rho}\frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

Where $w$ is displacement, $\bar{\rho}$ is mass per unit area, D is the flexural rigidity given for elastic plates by:

$$D = \frac{Yh^3}{12(1-\sigma^2)} \qquad (2)$$

Y is the Young's modulus and $\sigma$ is Poisson's ratio. A number of authors have definitions of D specifically for piezoelectric plates but all definitions provide similar results [8][9][10].

The general solution to (1) is given below, noting that displacement and stress are finite at r = 0 and the device is symmetrical about its diameter.

$$W(r,\theta) = (A_d J_d(\lambda) + B_d I_d(\lambda)) \cos d\theta \qquad (3)$$

Where $J_n$ and $I_n$ are the Bessel and modified Bessel functions of the first kind respectively, d is the number of nodal diameters (which is 0 for symmetrical vibrations), $A_d$ and $B_d$ are constants and $\lambda$ is the 'non-dimensional' frequency is related to frequency by (4).

$$\lambda^2 = \sqrt{\frac{\bar{\rho}}{D}} f r^2 \qquad (4)$$

Exact solutions to (2) can be found by applying boundary conditions, which in this case are for a completely free plate. This requires the bending moment ($M_r$) and Kelvin-Kirchhoff edge reaction ($V_r$) at r = a to be equal to zero. From this the frequency equation can be generated, as given in (5).

$$\frac{\lambda^2 J_d(\lambda) + (1-\sigma)\,\lambda J_d'(\lambda)}{\lambda^2 I_d(\lambda) - (1-\sigma)\,\lambda I_d'(\lambda)} - 1 = 0 \qquad (5)$$

Where $\lambda J_d'(\lambda)$ and $\lambda I_d'(\lambda)$ are given in Eq.6.

$$\begin{gathered}\lambda J_d'(\lambda) = -\lambda J_{d+1}'(\lambda) \\ \lambda I_d'(\lambda) = \lambda I_{d+1}'(\lambda)\end{gathered} \qquad (6)$$

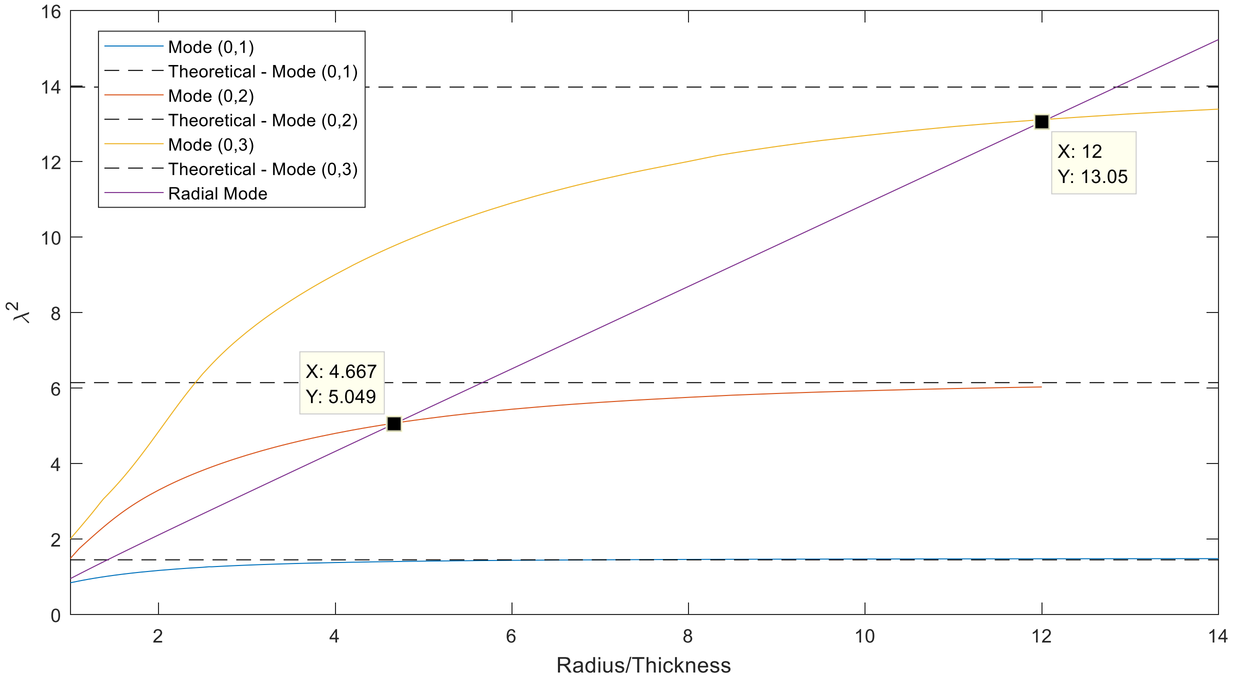


Fig 3 – Variation of 'non-directional' frequency $\lambda^2$ with the ratio of device radius to thickness

Solutions to (5) are reported by Leissa [5], shown in Table 1. The values of $\lambda^2$ can then be used to approximate the resonant frequency of each mode with (4).

Table 1

| c | $\lambda^2$ |
|---|---|
| 1 | 1.446 |
| 2 | 6.14 |
| 3 | 13.97 |

### 3.2. Finite Element Simulation

Due to the implications stated earlier, exact analytical solutions can only be solved in 2D. However, a PT is a 3 dimensional device which for most designs disagrees with Kirchhoff's thin plate assumptions, in particular the device will most likely not have a radius ten times that of its thickness. In order to accurately solve this type of problem, a FEM should be used. Another advantage of finite element analysis (FEA) is that a PT can modelled as a number of separate layers rather than 1 disk, further improving accuracy.

To investigate the relation between device geometry and resonant frequency of the flexural modes, a number of PZT-5H based devices will be simulated in COMSOL, with radii varying between 5 and 14mm and thicknesses between 1 and 5mm.

Simulations will be done with the output electrode(s) connected to ground. The resulting resonant frequencies will be extracted. The device dimensions have been chosen to reflect the size of devices that could be currently manufactured with commercially available PZT discs.

Initial simulations are done on PTs with an input layer thickness equal to the output layer thickness, $h_1 = h_2$ in Fig. 1 A further set of simulations will vary the input to output layer thickness ratio and observe the effect. A final set of simulations will be done on devices made up of 3-5 layers, to observe any shift in the resonant frequencies.

In order to be able to relate the results to other material types and to the results achieved through the analytical method, the resonant frequencies will be converted to the 'non-dimensional' frequency using (4). As the radial mode frequency is also extracted during the eigenfrequency simulation, it will be converted to 'non-dimensional' frequency and plotted for comparison. As the flexural frequencies are found in this method without Kirchhoff's assumptions, the non-dimensional frequency is expected to change as a function of how well the device agrees with the thin plate assumptions, thus the resulting values will be plotted against the ratio of radius/thickness.

# 4. Results

## 4.1. 2 Layer PT – Equal layer thickness

Fig. 3 shows the results of the first simulation, with the input and output layer thickness equal. As expected the FEM results match well with the analytical results for large values of radius/thickness, when the Kirchhoff assumption is valid. An important feature of this figure is the highlighting of the radius/thickness ratios which causes the radial mode to interact with the spurious modes. The figure also highlights the differences between the FEM and analytical results, in both cases the analytical approach shows the point at which two modes cross being at a radius/thickness ratio larger than the FEM approach. The same results are achieved when the output is driven and the input is short circuited to ground. This is due to the electrical symmetry caused by solving for free vibrations and so the input voltage, like the output terminal, is at 0V.

## 4.2. 2 Layer PT – Differing Layer thickness

One factor when designing a PT based converter is its ability to achieve ZVS, Foster, *et al*. [11], found that, by designing the ratio of input to output thickness to be less than $2/\pi$, ZVS can always be achieved for a matched load. In order to observe the effect of unequal layer thicknesses on resonant frequency, another parameter was included that changed the relative size of each layer between 10% and 50% of the total device thickness.

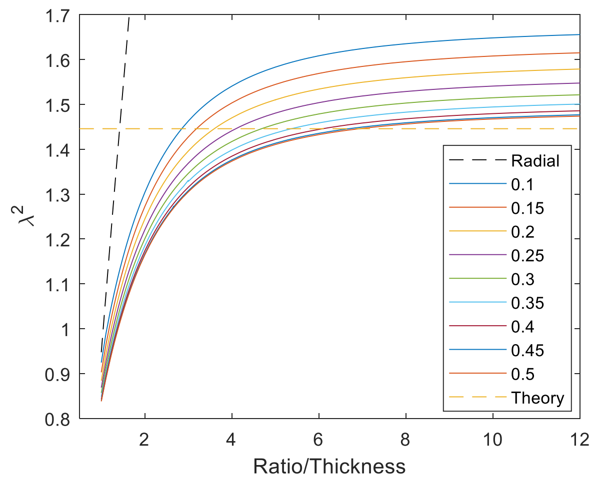


Fig 4 - Variation of $\lambda^2$ with the ratio of radius to thickness for devices for the first flexural mode with differing layer thicknesses

Figs. 4-6 show the how the curve for each mode changes slightly as one layer is made thicker than the second. An increase in resonant frequency is expected as the layer thicknesses become unbalanced, thus adjusting the optimal geometric size for avoiding interaction between modes.

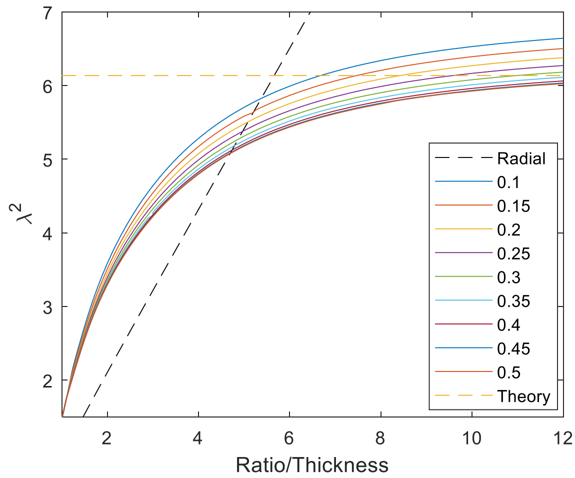


Fig 5 - Variation of $\lambda^2$ with the ratio of radius to thickness for devices for the first flexural mode with differing layer thicknesses

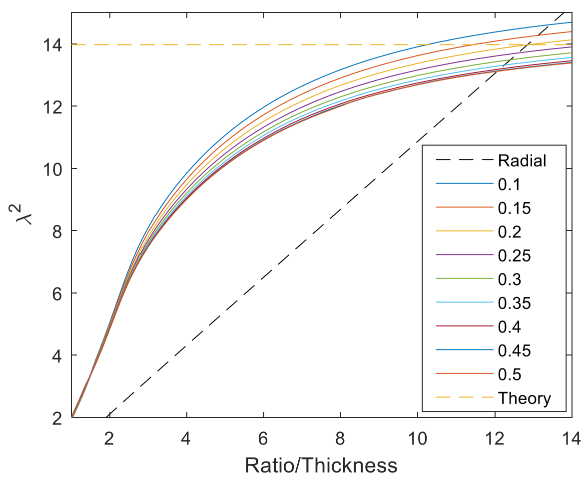


Fig 6 - Variation of $\lambda^3$ with the ratio of radius to thickness for devices for the first flexural mode with differing layer thicknesses

## 4.3 3-5 Layer PT's – Equal layer thicknesses

For most applications PTs will require three or more layers to provide an appropriate turns ratio. Fig. 7 shows how the resonant frequencies of each mode is affected by splitting the device into more layers. The opposite effect to changing the layer thicknesses in 2 layer designs is observed, leading to a decrease in resonant frequency.

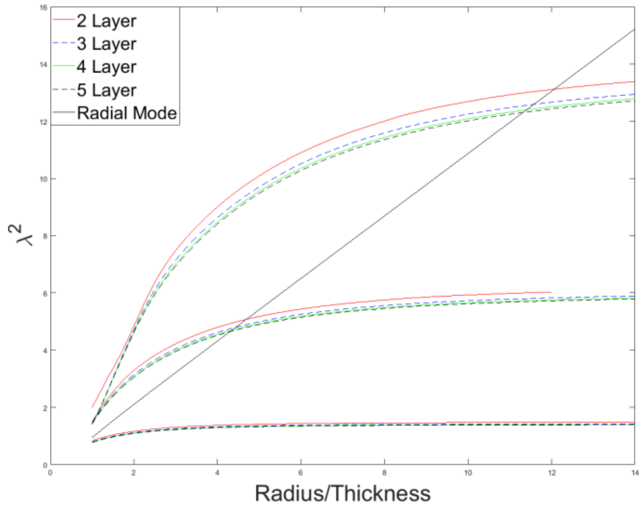


Fig 7 – Variation of $\lambda^2$ with ratio of radius to thickness for devices with varying number of layers

### 4.4 3-5 layer PTs – Differing layer thicknesses

Similar to the 2 layer designs, in most cases the layer thicknesses will need to be different and so this will have an effect on the resonant frequency of each mode. In order to visualise this effect a new metric must be created to encapsulate the relative thicknesses of the layers as a single value. Firstly, the curves obtained from these simulations showed the same shape as those observed in Figs. 3-7, and so each change in layer thickness enlarges each curve by a scaling factor. Results show that the scale factor changes as a function of the variance in the layer thicknesses.

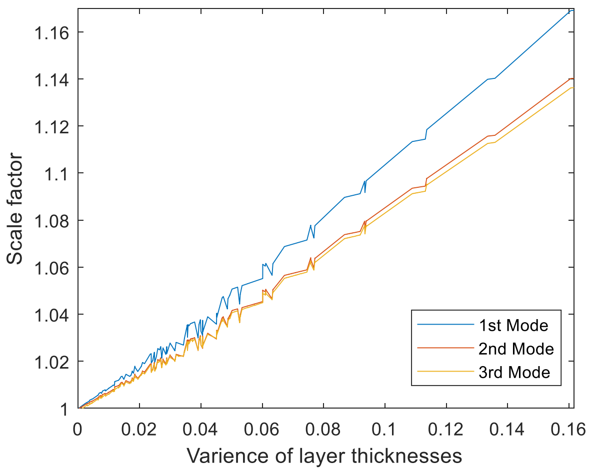


Fig 8 – Variation of $\lambda^2$ scale factor with variance in layer thickness for 3 layer devices

The results of the simulation, shown in Fig. 8, show this general trend. The scale factor changes linearly with layer thickness variance, so increasing the imbalance of the layer thicknesses causes an increase in the resonant frequency, a similar trend to that seen in 2 layer designs.

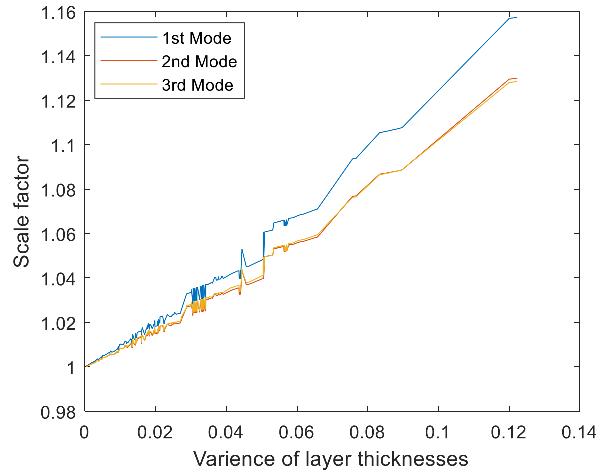


Fig 9 - Variation of $\lambda^2$ scale factor with variance in layer thickness for 4 layer devices

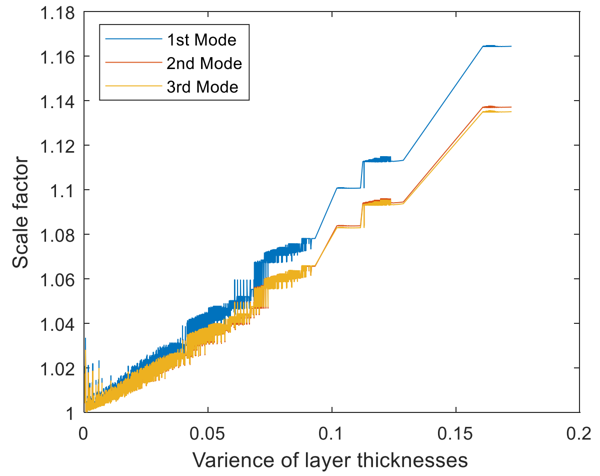


Fig 10 - Variation of $\lambda^2$ scale factor with variance in layer thickness for 5 layer devices

Figs. 9 and 10 highlight the same effect in 4 and 5 layer designs respectively.

## 5. Implications of mode interaction

In order to investigate the effects of spurious modes on radial vibration, a frequency domain study was performed on a PT with a radius of 9mm and a thickness of 2.2mm. From Fig. 3 we can observe that this will place the resonant frequency of the 2nd flexural mode and the radial mode in close proximity.

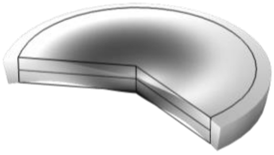

Fig 11 – Displacement of a PT with radius of 9mm and thickness of 2.2mm driven at a frequency of 110kHz

Fig. 11 shows the displacement of device when operated at a frequency just above radial resonance. The results show that even with the two modes occurring 10 kHz apart, there is still an effect on the quality of the radial vibration. This will relate to poor performance as the coupling factor will be reduced and damping will be increased.

A final factor to consider is the effect spurious modes have on the electrical operation of the device, specifically the efficiently. PT based power converters are known to be highly efficient but to achieve this high intrinsic PT efficiency is vital. To observe this the Mason equivalent circuit was extended to include a spurious resonant branch and related transformer as shown by Lin [12].

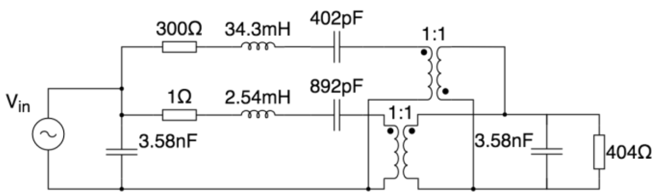


Fig 12 – Extended Mason equivalent circuit of a PT with radius of 9mm and thickness of 3mm. The radial resonance is at 105 kHz, so the circuit will be driven at 115kHz during operation. Damping values were chosen from real PT measurements on a similar device. The load was chosen from matched condition

The circuit is shown in Fig. 12, with equivalent parameters from a device with a radius of 9mm and a thickness of 3mm. A frequency domain study was performed and the efficiency of the device was extracted over a wide range of frequencies.

Fig. 13 shows the resulting curve. The main feature of the graph is the sharp drop in efficiency at 33kHz. This loss of efficiency is due to a filtering affect caused by a combination of the two modes. At that particular frequency the current from the two branches is equal in magnitude and opposite

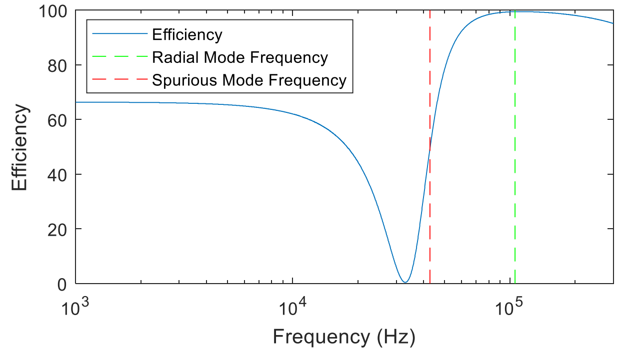


Fig. 13 – Efficiency of PT over a range of frequencies, plotted with spurious and radial resonant frequency for comparison

in phase, thus the current circles through the branches and output current approaches zero. In order to avoid this condition occurring in normal operation, an equation for the frequency that this condition occurs at was generated and is presented in (7).

$$\boldsymbol{\omega} \approx \pm \frac{\sqrt{-(\boldsymbol{C1C2}(\boldsymbol{L1}-\boldsymbol{L2})(\boldsymbol{C1}-\boldsymbol{C2}))}}{\boldsymbol{C1C2}(\boldsymbol{L1}-\boldsymbol{L2})} \quad (7)$$

Equation (7) explains that the frequency that this condition occurs at is a function of the inductance and capacitance in each branch. To fully observe this effect, a second circuit simulation was performed in Simulink to observe how efficiency changes with resonant frequency and relative ratio of inductance to capacitance (L/C) of the spurious mode compared to the radial mode. The spurious mode resonant frequency was moved between 10kHz and 300kHz. The ratio of relative spurious to radial branch L/C was changed between 1 to 1500 times that of the radial mode, this was chosen after a number of devices were simulated and this range fully covers all typical values. The frequency of operation ($f_{OP}$) was 115kHz.

## 6. Results

Fig. 14 shows how the efficiency loss of the PT increases with proximity of the spurious resonant frequency to the radial resonant frequency. Another interesting feature that this figure shows is that for similar magnitudes of L/C in both branches, the spurious mode has to be at a frequency much further away from the radial mode in order to keep efficiency high. An important consideration when analysing the results in Fig. 14 is that the graph is largely dependent on the value of damping in the spurious mode.

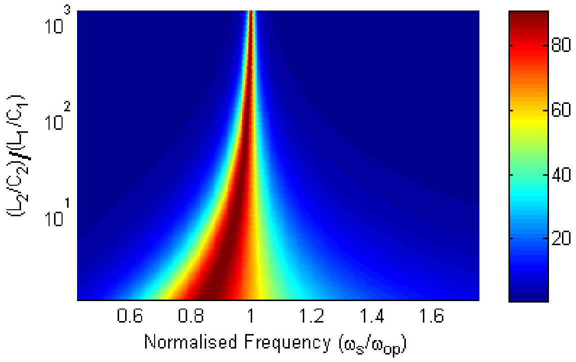


Fig 14 – Efficiency loss (100-Efficiency) (%) with variation in normalised spurious resonant frequency and ratio of spurious inductance/capacitance relative to the radial mode

Fig. 15 shows how the efficiency loss of the transformer varies with proximity and damping. As the damping increases, similar to low values of relative L/C in Fig. 14, the further away the resonant frequency of the spurious mode needs to be from the radial mode in order to keep efficiency high. A final point is that if the spurious mode is damped to such a level that the current though it is minimal, then the apparent effect of this mode is highly reduced and the overall efficiency is not affected by this spurious mode.

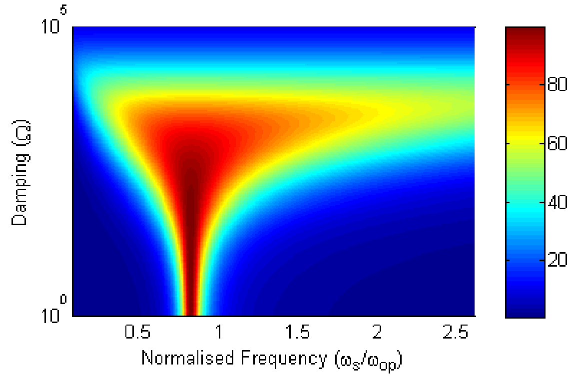


Fig 15 - Efficiency loss (100-Efficiency) (%) with variation in normalised spurious resonant frequency and spurious mode damping with ratio of spurious inductance/capacitance relative to the radial mode of 1

## 7. Discussion and guidelines for designers

The analysis presented in this paper highlights some of the important factors to consider when designing PTs in order to reduce the effect of spurious modes. Firstly, the best method of reducing the effect of spurious modes is to design the transformer such that the resonant frequencies of spurious modes are as far from the radial mode as possible. Avoiding radius and thickness values that give a ratio of close to approximately 1, 4.667 or 12 is necessary. If values of radius and total thickness are chosen such that their ratio is approximately 3 or 8.5, the radial mode frequency will occur directly in-between two unwanted modes.

In some cases, this will not be possible and so a number of other factors need to be considered. Ideally minimal or maximal damping is required in the spurious vibration, limiting the range of frequencies affected by this mode. Damping can be reduced through a number of methods such as, high Q materials, quality manufacturing and appropriate mounting. A final consideration is to design the device to have a much larger L/C ratio in the spurious mode than in the radial mode. This can be achieved by choosing a design that has a large ratio of radius to thickness and is also large in radius. For example, a device with a radius/thickness ratio of 1 has a relative L/C ratio much less than a device with a radius/thickness ratio of 5. Also a device with a radius of 5mm and thickness of 1mm has a lower relative L/C ratio that a device with a radius of 15mm and thickness of 3mm.

## 8. Conclusions

The work presented in this paper is an initial analysis of where spurious modes occur in piezoelectric transformers and investigates the effect they have on performance. It has been shown that by careful geometric design, the effect of spurious modes can be reduced or in some cases eliminated. A number of design rules have been presented, as to reduce the effect of spurious modes. Although, due to the number of independent parameters involved, it isn't possible without FEA to determine if the effect of a spurious mode is completely suppressed. In almost all cases designing a transformer to the rules presented, will prevent deleterious effects.

## 9. References


[1] E. L. Horsley, M. P. Foster, and D. A. Stone, 'State-of-the-art Piezoelectric Transformer technology', in *2007 European Conference on Power Electronics and Applications*, 2007, pp. 1–10.

[2] K. Brebøl, ‘Piezoelectric transformer’, US6707235B1, 16-Mar-2004.

[3] G. Kirchhoff, ‘Ueber die Schwingungen einer kreisförmigen elastischen Scheibe’, *Ann. Phys.*, vol. 157, no. 10, pp. 258–264, Jan. 1850.

[4] E. Ventsel and T. Krauthammer, *Thin plates and shells: theory, analysis, and applications*. New York: Marcel Dekker, 2001.

[5] A. W. Leissa, ‘Vibration of Plates’, OHIO STATE UNIV COLUMBUS, 1969.

[6] P. C. Y. Lee, R. Huang, X. Li, and W.-H. Shih, ‘Vibrations and static responses of asymmetric bimorph disks of piezoelectric ceramics’, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 47, no. 3, pp. 706–715, May 2000.

[7] N. T. Adelman and Y. Stavsky, ‘Flexural–extensional behavior of composite piezoelectric circular plates’, *J. Acoust. Soc. Am.*, 1980.

[8] R. D. Mindlin, ‘Forced Thickness-Shear and Flexural Vibrations of Piezoelectric Crystal Plates’, *J. Appl. Phys.*, vol. 23, no. 1, pp. 83–88, Jan. 1952.

[9] F. Sammoura and S. G. Kim, ‘Theoretical modeling and equivalent electric circuit of a bimorph piezoelectric micromachined ultrasonic transducer’, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 59, no. 5, pp. 990–998, May 2012.

[10] Y.-H. Huang, C.-C. Ma, and Z.-Z. Li, ‘Investigations on vibration characteristics of two-layered piezoceramic disks’, *Int. J. Solids Struct.*, vol. 51, no. 1, pp. 227–251, Jan. 2014.

[11] M. P. Foster, J. N. Davidson, E. L. Horsley, and D. A. Stone, ‘Critical Design Criterion for Achieving Zero Voltage Switching in Inductorless Half-Bridge-Driven Piezoelectric-Transformer-Based Power Supplies’, *IEEE Trans. Power Electron.*, vol. 31, no. 7, pp. 5057–5066, Jul. 2016.

[12] R.-L. Lin, ‘Piezoelectric Transformer Characterization and Application of Electronic Ballast’, *PhD Diss.*, Nov. 2001.